

MOJZA

AS Level

Mathematics

Paper 1 Notes

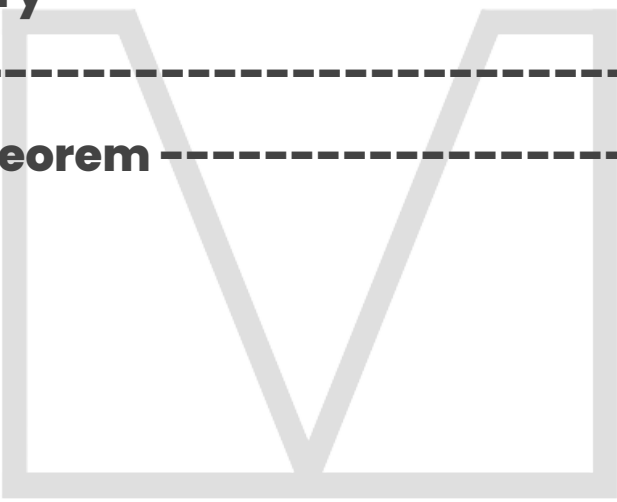
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BY TEAM MOJZA

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Quadratics

- Types Of Forms:

Standard Form	$ax^2+bx+c = 0$	a = nature of curve c = y-intercepts
Completed Square Form	$a(x-b)^2+c = 0$	(b,c) = vertex point
Root Form	$a(x-b)^2(x-c)^2 = 0$	$(b,0)(c,0)$ = x-intercepts

- Intersection:

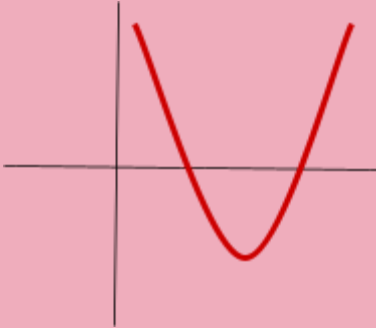
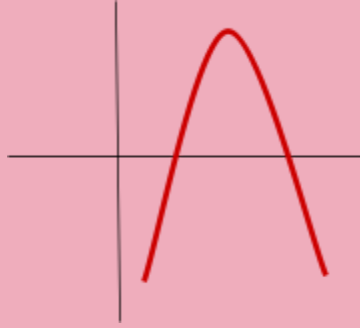
- When a Line and Curve intersect with each other, their 'y' and 'x' coordinates are the same.
- To find out the point of intersection, both equations are equated.
- The number of points depends upon the discriminant of the quadratic formula formed.

- Discriminant:

$b^2-4ac>0$	$b^2-4ac=0$	$b^2-4ac<0$
Two distinct real roots/ Two points of intersection	Two same real roots/ Two same points of intersection	No real roots/ No point of intersection

- Nature:

- To convert from standard to completed square: Use **Factorization** method.
- "**a**" in every equation tells about **Nature Of Curve**:

Minimum Nature - 'a' Positive	Maximum Nature - 'a' Negative
 <p data-bbox="219 682 560 766">Opens Upwards Has Minimum Turning Point</p>	 <p data-bbox="852 682 1209 766">Opens Downwards Has Maximum Turning Point</p>

Types Of Questions In Exam:

- Completing The Square
- Solving The Inequality
- Solutions To A Graph



Coordinate Geometry

- Formulas:

- Distance Between Two Points: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
- Midpoint Of Two Points: $((x_1 + x_2)/2, (y_1 + y_2)/2)$
- Gradient Of A Line: $[(y_2 - y_1) / (x_2 - x_1)]$

- Equation Of A Line:

- $y - y_1 = m(x - x_1)$
- Where y_1 and x_1 are points on line, and m is the gradient.
- A perpendicular line gradient will be the negative reciprocal of the gradient of line.

- Equation Of A Circle:

- $(x - a)^2 + (y - b)^2 = r^2$
- Where (a,b) is the centre of the circle and r is radius.
- Example: If $(2,5)$ is the centre of a circle with radius 6 cm. The equation would be: $(x - 2)^2 + (y - 5)^2 = 36$

- Circle Properties:

- Tangent is a line which touches a circle at one point forming a 90° with the radius.
- Radius is the maximum distance from the centre of the circle to the circumference.
- Diameter always makes a right angle triangle in circle
- To find out equation of circle using 2 points:
 - Find the midpoint first using formula, points of diameter touching edges
 - Find the length of diameter and divide it by 2
 - Substitute the values in formula

Types Of Questions In Exam:

- Equation Of A Line
- Equation Of Perpendicular Bisector Of A Line
- Finding Unknown Points Of A Shape
- Using Expanded Formula Of Circle
- Circle Properties



Circular Measure

- Degrees And Radians:

- There are 2 ways to measure an angle i.e degree ($^{\circ}$) and radians (rad)
- We mostly use 'pi' in radians
- $1 \text{ Pi} = 180^{\circ}$
- One circle is 360° or 2π

90°	180°	270°	360°
$\frac{1}{2} \pi$	π	$\frac{3}{2} \pi$	2π

- Area Of Sector:

- Degree formula - $\frac{\theta^{\circ}}{360} \times \pi r^2$
- Radian formula - $\frac{1}{2} \times r^2 \theta$

- Perimeter Of Sector (Arc Length):

- Degree formula - $\frac{\theta^{\circ}}{360} \times 2\pi r$
- Radian formula - $r \times \theta$

Types Of Question In Exam:

- Finding Arc Length Of Shaded Region
- Finding Area Of Shaded Region

Series

- Arithmetic Progression:

- A constant term is added or subtracted each time.
- n^{th} Term = $\mathbf{a+(n-1)d}$
- Sum = $\mathbf{n/2 (2a+(n-1)d)}$

- Geometric Progression:

- A constant term is multiplied or divided each time any term except 0 or 1
- N^{th} term = $\mathbf{a \times r^{n-1}}$ (where r is common difference)
- Sum = $\mathbf{a (1 - r^n) / (1 - r)}$
- Sum to infinity = $\mathbf{a/1-r}$

- Exam Tip To Keep In Mind:

	Arithmetic	Geometric
1st Term	a	a
2nd Term	a+d	ar
3rd Term	a+2d	ar ²
4th Term	a+3d	ar ³
5th Term	a+4d	ar ⁴
10th term	a+9d	ar ⁹

Types Of Question In Exam:

- Formula Based
- Scenario Based
- Ap And Gp



Differentiation

- How To Differentiate:

- If a number with no Variable, it will become 0
- If a coefficient with x^1 , then x value is = 1 - coefficient will remain the same (x removed)
- If a term with $x^{n>1}$ - Power rule is applied (coefficient is multiplied by power, power is reduced by 1 and term is multiplied with the differentiation of the variable)
- y is now differentiated into dy/dx

- Gradient:

- Differentiating an equation leads us to the equation of gradient of the curve at a point
- To find the gradient at a particular point of curve - substitute the x coordinate into the dy/dx equation
- The gradient at the maximum or minimum point of curve is always 0
- To find the maximum or minimum point coordinate - we can equate dy/dx to 0 and solve

- Nature:

- To find whether the curve is a maximum or minimum nature -
- We can simply differentiate the dy/dx - and get d^2y/dx^2
- And then substitute the stationary point coordinate - if answer positive - minimum otherwise maximum

- Increasing And Decreasing Function:

- IF $dy/dx > 0$ then the curve is an increasing function
- IF $dy/dx < 0$ then the curve is an decreasing function

- Scenario Based Questions:

- These questions include 3 terms; 2 different variables and 1 connecting equation
- Mostly in the form of rate - eg: dx/dt , dy/dt
- To solve them, a chain rule is first formed and then values are substituted

Types Of Questions In Exam:

- Equation Of Tangent
- Stationary Points
- Rate Change



Integration

It is a the reverse working of differentiation

- How To Integrate:

- If a term has any variable - **power rule is applied** (power is increased by 1, term is divided by new Power, and also divide by the differentiated value of variable)
- If a term has **no** variable - attach an X to it
- dy/dx is changed to y

- Equation Of Line:

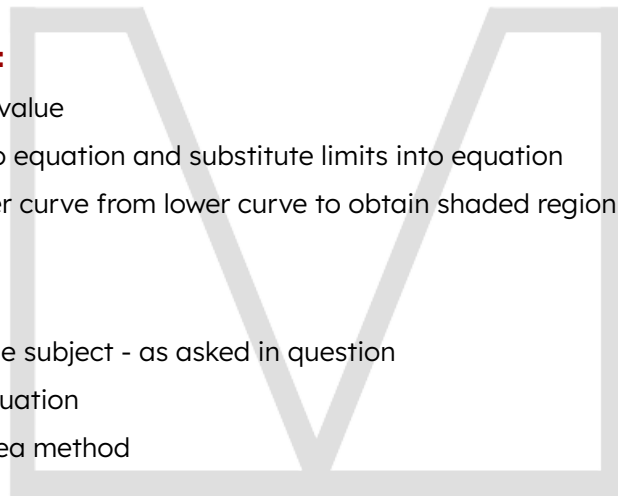
- Integrate the value
- Add a '+ c' to the integrated equation
- Substitute the coordinates of curve to find the constant
- Rewrite integrated equation with the constant value

- Area Under Curve:

- Integrate the value
- Add x-limits to equation and substitute limits into equation
- Subtract upper curve from lower curve to obtain shaded region area

- Volume Of Curve:

- Make y or x the subject - as asked in question
- Square the equation
- Repeat the area method



Types Of Questions In Exam:

- Equation Of Curve
- Area Under Curve
- Volume Under Curve

Trigonometry

A Level trigonometry is different than O'Level trigonometry
 We should be careful while choosing radian or degree mode

- Identities:

- $\sin^2\theta + \cos^2\theta = 1$
- $\sin^2\theta = 1 - \cos^2\theta$
- $\cos^2\theta = (1 - \sin^2\theta) = (1 + \sin\theta)(1 - \sin\theta)$
- To bring an addition/subtraction expression to 1 term - Take out LCM
- To bring into 1 common trig ratio, use identity

- Base Angle:

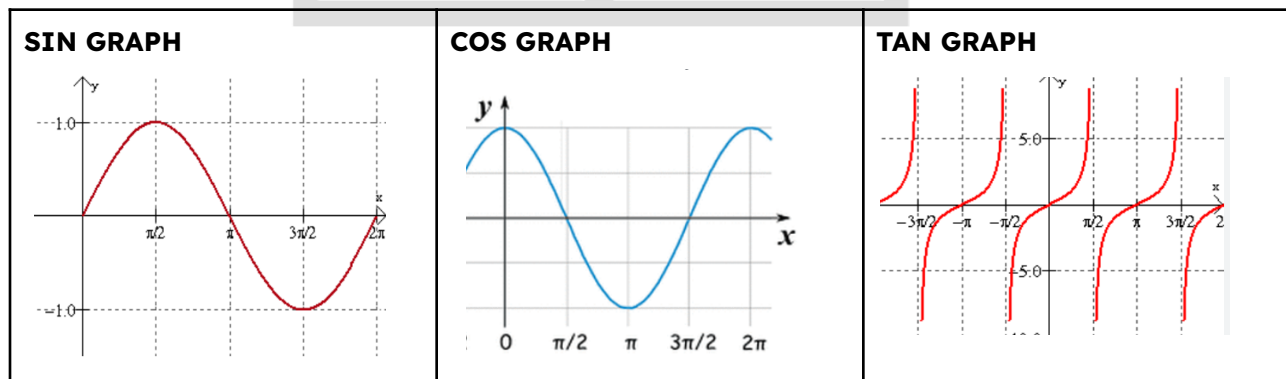
- It is the minimum angle from x-axis, can be used to find solutions of a trig ratio
- Basic angle is between $0^\circ - 90^\circ$
- SIN, COS, TAN are all positive in the first quadrant

- Range Check:

- Angles are repeated after one 360° rotation
- If the questions asks for angles between $0^\circ - 720^\circ$, angle will rotate twice
- If the question asks for angles between $0^\circ - 180^\circ$, angles more than 180 won't be considered.

- Graphs:

- $a\sin(b\theta) + y$ where a=amplitude, b=no.of waves, y=constant line
- Tan completes 1 wave in 180, whereas sin and cos complete 1 wave in 360



Types Of Question In Exam:

- Prove The Identity
- Find The Angle
- Graph



Functions

- Terms:

- Y coordinate - $f(x)$ / range / image
- X coordinate - (x) / domain

- Inverse Functions:

- For an inverse function to exist, It should be ONE TO ONE

- Composite Function:

- If $g(f(x))$: function F will be substituted as (x) of function G
- Ranges and domains in composite function will be followed by inner function.

- Transformation Function:

- $f(x) + a$ = transform 'a' units up/down
- $f(x+a)$ = transform '-a' units left/right
- $mf(x)$ = enlarging by factor 'm' parallel to y-axis
- $f(mx)$ = enlarge by factor $1/m$ parallel to x-axis

- Increasing Decreasing Functions:

- Covered in differentiation !

Types Of Question In Exam:

- With Quadratics
- With Trigonometry
- Composite Functions
- With Differentiation

Binomial Theorem

- Binomial means 2 terms - binomial expansion is the expansion of 2 terms.
- Comes for around 4-5 marks in exam
- “ nC_r ” button is used on calculator - shift + divide
- Used to expand higher power brackets
- Total terms are always (n+1)

- Formula:

- $(a+b)^n = nC_r \times b^r \times a^{(n-r)}$
- ‘r’ is always “ term number - 1 ”
- ‘n’ is the power

- Finding First Few Terms:

$$(a+b)^4 = {}^4C_0b^0a^{4-0} + {}^4C_1b^1a^{4-1} + {}^4C_2b^2a^{4-2} + {}^4C_3b^3a^{4-3} + {}^4C_4b^4a^{4-4}$$

$$= a^4 + 4ba^3 + 6b^2a^2 + 4b^3a + b^4$$

- When Finding Specific Term Numbers:

- If variable x has power of 1 then r is equal to the term number asked
- If variable has power > 1 ‘x² or x³ = find out r by formula

- Multiplying Two Brackets:

- Do not expand the whole bracket- rather just the coefficients needed.
- Do not miss multiplying the signs
- Remember to multiply the variables as well (powers add up)

- Remember:

- ${}^nC_n = 1$
- ${}^nC_0 = 1$
- Term independent of x = term with x⁰

Types Of Questions In Exam:

- Expand First Few Terms
- Find Coefficient Of Specific Terms



A Note from Mojza

These notes for AS Level Mathematics (9709) have been prepared by Team Mojza, covering the content for AS Level 2022-24 syllabus. The content of these notes has been prepared with utmost care. We apologise for any issues overlooked; factual, grammatical or otherwise. We hope that you benefit from these and find them useful towards achieving your goals for your Cambridge examinations.

If you find any issues within these notes or have any feedback, please contact us at support@mojza.org.

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